The DUAL DEA MODEL

For any linear program (LP) it is possible to formulate a partner LP using the same data, and the solution to either the original LP (the primal) or the partner (the dual) provides the same information about the problem being modelled. DEA is no exception to this. The dual model is constructed by assigning a variable (dual variable) to each constraint in the primal model and constructing a new model on these variables. This is shown below.

The first thing to note is that the primal model has n + t + m + 1 constraints whilst the dual model has m + t constraints. As n, the number of units, is usually considerably larger than t + m, the number of inputs and outputs, it can be seen that the primal model will have many more constraints than the dual model. For linear programs in general the more constraints the more difficult a problem is to solve. Hence for this reason it is usual to solve the dual DEA model rather than the primal.

From the theory of linear programming it is known that the values of the dual variables as a result of solving a dual model are identical to the shadow prices in the primal model. The dual variables Lambda(j) are thus also the shadow prices related to the constraints limiting the efficiency of each unit to be no greater than 1. It is also known that where a constraint is binding, a shadow price will be positive normally and where the constraint is non-binding the shadow price will be zero. In the solution to the primal model therefore a binding constraint implies that the corresponding unit has an efficiency of 1 and there will be a positive shadow price or dual variable. Hence positive shadow prices in the primal, or positive values for the Lambda(j) ‘s in the dual, correspond to and identify the peer group for any inefficient unit.

ADVANTAGES OF DEA

No need to explicitly specify a mathematical form for the production function.

Proven to be useful in uncovering relationships that remain hidden for other methodologies.

Capable of handling multiple inputs and outputs.

Capable of being used with any input-output measurement.

The sources of inefficiency can be analysed and quantified for every evaluated unit.

DISADVANTAGES OF DEA

Results are sensitive to the selection of inputs and outputs (Berg 2010).

You cannot test for the best specification (Berg 2010).

The number of efficient firms on the frontier tends to increase with the number of inputs and output variables (Berg 2010).

A desire to Improve upon DEA, by reducing its disadvantages or strengthening its advantages has been a major cause for many discoveries in the recent literature.

The currently most often DEA-based method to obtain unique efficiency rankings is called cross-efficiency.

Limitations of Data Envelopment Analysis

DEA is a great technique but it has its limitations. You must understand that DEA is like a black box. Since the weights that are used in the effectiveness ratio of each record are different, trying to explain how and why each score was calculated is pointless. Usually we focus on the ranking of the records rather than on the actual values of the effectiveness scores. Also note that the existence of extremums can cause the scores to have very low values.

Have in mind that DEA uses linear combinations of the features to estimate the ratios. Thus if combining them linearly is not appropriate in our application, we must apply transformations on the features and make them possible to be linearly combined. Another drawback of this technique is that we have to solve as many linear programming problems as the number of records, something that requires a lot of computational resources. Another problem that DEA faces is that it does not work well with high dimensional data.

CONCLUSIONS

DEA is a novel approach to relative efficiency measurement where there are multiple incommensurate inputs and outputs. If a suitable set of measures can be defined DEA provides an efficiency measure not relying on the application of a common weighting of the inputs and outputs. Additionally the method identifies peer units and targets for inefficient units. A number of issues arising from the application of DEA have also been addressed.